ECON 7010 - Macroeconomics I

Fall 2015

Notes for Lecture #11

Today:

- Models of economic fluctuations with money - proportional and non-proportional transfers
- Models of economic fluctuations with asymmetric info

Let's work with a specific example...

Proportional Transfers

- Money supply: $M_{t+1} = M_t x_{t+1}, \ x_{t+1} \sim f(\cdot)$, iid
  - Money held in period $t+1$= money held in period t multiplied by x_{t+1} (the increase in money holdings comes from money printed by gov't and given to the agents)
- Optimization of generation t:
  - $\max_{n_t} E_{(x_{t+1},p_{t+1}|s_t)} u\left(\frac{p_t n_t x_{t+1}}{p_{t+1}}\right) - g(n_t)$
    * $s_t \equiv$ state in period t, a list of variables telling you where you currently are
    * Note that the expectation of p_{t+1} is defined over a distribution of prices that is endogenous, we solve for it when we find the Rational Expectations Equilibrium
    * In the equation above, we substituted in for c_{t+1} with the B.C.: $c_{t+1} = \frac{p_t n_t x_{t+1}}{p_{t+1}}$
  - the FOC is: $E_{(\cdot)}\left[\frac{p_t x_{t+1}}{p_{t+1}} u'\left(\frac{p_t n_t x_{t+1}}{p_{t+1}}\right)\right] = g'(n_t)$ (so, similar to what we had before, but now with the expectations operator)
- Market clearing: $M_t = p_t n_t$, $\forall t$ (money market clearing $\Rightarrow$ goods market clears because only 2 markets and Walras' Law)
  - Note, we are using p_t and not π_t for the dollar price of goods
- Stationary Rational Expectations Equilibrium (SREE):
  - Recall that an equilibrium is a sequence of allocations ($\{n_t\}_{t=1}^{\infty}$) and prices ($\{p_t\}_{t=1}^{\infty}$).
  - Consider these as two functions:
    1. $n_t = n(s_t)$
    2. $p_t = p(s_t)$
  - These functions are consistent with:
    1. Individual optimization
       * Use $p(s_t)$ to calculate expected values
       * $n(s_t)$ is the decision rule
    2. Market clearing
       * $M_t = p(s_t) * n(s_t), \ \forall t$
  - Note how p and n are functions of the state variables
  - p and n are stationary - they do not depend upon t
- A (Good) Guess (using our intuition about such things as neutrality of money, etc...)
  - $s_t = (M_{t-1}, x_t)$
  - $n_t = n(M_{t-1}, x_t) = \bar{n}$, where $\bar{n}$ is a constant b/c of the neutrality of money, $\bar{n}$ also solves the $u'(n) = g'(n)$ FOC

- $p_t = p(M_{t-1}, x_t) = QM_{t-1}x_t,\ Q \equiv$ unknown (it's a factor of proportionality b/c neutrality of money; prices proportional to money supply), reason also for this guess is that $M_t = M_{t-1}x_t$
    - from market clearing: $M_t = p(s_t) * n(s_t) = p(s_t)\bar{n} \Rightarrow M_t = Q\underbrace{M_{t-1}x_t}_{=M_t}\bar{n} \Rightarrow 1 = Q\bar{n} \Rightarrow Q = \frac{1}{\bar{n}}$
- Now let's verify that these guesses work.
    - Rewrite the maximization problem from earlier with our guess at s_t, n_t, p_t.
    - $\Rightarrow$ optimization of generation t:
    - $\max_{n_t} E_{x_{t+1}} u\left(\frac{p_t n_t x_{t+1}}{QM_t x_{t+1}}\right) - g(n_t)$
    - Where you can cancel out much of the fraction above since $p_t = Q\underbrace{M_{t-1}x_t}_{M_t}$ (from definition of the money supply)
    - $\Rightarrow$ can rewrite as: $\max_{n_t} u(n_t) - g(n_t)$
    - $\Rightarrow$ FOC: $u'(n_t) = g'(n_t)$, $n_t = \bar{n}$ solves this... then get prices from MC: $Q = \frac{1}{\bar{n}}$ and $p_t = QM_t = \frac{M_t}{\bar{n}}$
    - THERE does exist an SREE!
    - There may be other SREE and there are certainly other REE (e.g. where constants vary with time, but not state: $n(s_t) = \bar{n}_t$ and $p(s_t) = Q_t M_{t-1} x_t$)

- <u>Another guess</u>:
    - $p_t = QM_{t-1} \rightarrow$ says prices don't respond to x_t, Q unknown
    - $n_t = zx_t \rightarrow$ says labor supply does respond to x_t
    - Market clearing: $M_t = p_t n_t = QM_{t-1} * zx_t = QzM_{t-1}x_t = QzM_t \Rightarrow 1 = Qz$
    - For individual optimization, use guess in FOC:
        - $E\{\frac{p_t x_{t+1}}{p_{t+1}} u'\left(\frac{p_t n_t x_{t+1}}{p_{t+1}}\right)\} = g'(n_t)$
        - $\Rightarrow E\{\frac{QM_{t-1}x_{t+1}}{QM_t} u'\left(\frac{M_{t+1}}{QM_t}\right)\} = g'(n_t)$
        - $\Rightarrow E\{\frac{x_{t+1}}{x_t} u'\left(\frac{x_{t+1}}{Q}\right)\} = g'(\frac{x_t}{Q})$
        - But this can't hold $\forall\ t$, b/c if $x_t \uparrow$, rhs $\uparrow$ ($g'' > 0$) and lhs $\downarrow$ (b/c x_t in denominator)
        - $\Rightarrow$ This guess is not a solution - i.e. this is not an SREE!

BACK to general case...
<u>OG Model with Production</u>: SREE

- <u>Individual optimization</u>:
    - $\max_n E_{(x_{t+1}, p_{t+1}|\cdot)} u\left(\frac{p_t n_t x_{t+1}}{p_{t+1}}\right) - g(n_t)$ (written this way, we have uncertain, but proportional shocks to money)
    - $\Rightarrow$ FOC: $E\frac{p_t x_{t+1}}{p_{t+1}} u'(\cdot) = g'(n)$
- <u>Market Clearing</u>:
    - $\frac{M_t}{p_t} = n_t,\ t = 1, 2, ...$
    - We also know, from how we defined the shocks, that $M_{t+1} = M_t x_{t+1}$
- <u>SREE</u>:
    - let s_t denote the state in period t
    - Define two functions: $p_t = p(s_t)$, $n_t = n(s_t)$

- $p(s_t)$ and $n(s_t)$ jointly satisfy individual optimization and market clearing, for all s_t
    * The above functions (and thus the REE) is stationary because though the state (s_t) changes (hence the index), the functions $n(\cdot)$ and $p(\cdot)$ don't (hence no subscripts on the functions)

- Solving for the SREE:
  - Substitute the market clearing condition into the FOC to get an expression in $n(s_t)$ alone:
  - MC says: $p(s_t) = \frac{M_t}{n(s_t)}$
  - $\Rightarrow \frac{p(s_t)x_{t+1}}{p(s_{t+1})} = \frac{M_t}{n(s_t)} * x_{t+1} * \frac{n(s_{t+1})}{\underbrace{M_{t+1}}_{=M_t x_{t+1}}} = \frac{n(s_{t+1})}{n(s_t)}$
  - Now use this in the FOC (together with BC that says, $c_{t+1} = \frac{p_t x_{t+1} n(s_t)}{p_{t+1}} = \frac{n(s_{t+1})n(s_t)}{n(s_t)} = n(s_{t+1})$):
  - $\Rightarrow$ FOC now: $E_{s_{t+1}|s_t} n(s_{t+1})u'(n(s_{t+1})) = n(s_t)g'(n(s_t)), \forall s_t$
  - NOTE that there is no expectation for p_{t+1}, this is b/c of RE and the assumption that the agents all forecast the equilibrium price, adjust for equilibrium with that, then the prophecy is fulfilled b/c they all do it.
  - Note also that the LHS depends on s_t because of the expectation
  - We can rewrite the above with s' and s:
  - $E_{s'|s} n(s')u'(n(s')) = g'(n(s))n(s), \forall s$
  - $n(\cdot)$ is the unknown object we want to solve for with this difference equation.
  - We then use $n(s)$ to solve for $p(s)$ using the market clearing condition ($p(s) = \frac{M}{n(s)}$)

- We can show that there exists an SREE where money is neutral: This is just what we did for the guess before to prove this...

- Let σ_x be the std dev of x. What value of σ_x does society prefer?
  - Social welfare is $u(\bar{n}) - g(\bar{n})$, but since we assume RE, prices are neutral, so the std. dev. doesn't matter - there is not change in labor supply as prices change

- We had a good guess and a bad guess, but the fact is that with RE and proportional transfers, money is always going to be neutral!

Non-proportional transfers:

- Usual model of OG with production

- $M_{t+1} = M_t(\sigma + 1)$ This is the aggregate law of motion for M_t. (It says that there is no uncertainty to the increase)
  - σ is the rate of growth in the money supply

- Individual budget constraint:
  - $c_{t+1} = \frac{p_t n_t + \gamma_{t+1}}{p_{t+1}}$ - note how money transfers are not proportional to money held
  - Where γ_{t+1} is the lump sum given to the agent, $\Rightarrow M_{t+1} = M_t + \gamma_{t+1} \Rightarrow \gamma_{t+1} = M_{t+1} - M_t$
  - By the law of motion for the money supply, $M_{t+1} = M_t(\sigma+1) \Rightarrow M_{t+1} - M_t = \sigma M_t \Rightarrow \gamma_{t+1} = \sigma M_t$
  - Thus σ is the percentage increase in M_t and γ_{t+1} is the actual (level) increase in M_t

- Individual optimization:
  - $\max_{n_t} u\left(\frac{p_t n_t + \gamma_{t+1}}{p_{t+1}}\right) - g(n_t)$

- FOC is: $\underbrace{\frac{p_t}{p_{t+1}}}_{\text{the real wage}} u'\left(\frac{p_t n_t + \gamma_{t+1}}{p_{t+1}}\right) = g'(n_t)$

- Market clearing:
  - $\frac{M_t}{p_t} = n_t, \forall t \Rightarrow \frac{p_t}{p_{t+1}} = \frac{M_t}{n_t} * \frac{n_{t+1}}{\underbrace{M_{t+1}}_{=M_t(1+\sigma)}} = \frac{n_{t+1}}{n_t(1+\sigma)}$
  - Plugging this into the B.C. we get: $c_{t+1} = \frac{p_t n_t + \gamma_{t+1}}{p_{t+1}} = \frac{M_t(1+\sigma)}{M_{t+1}/n_{t+1}} = n_{t+1}$

- Equilibrium:
  - Satisfies Ind opt and MC
  - Substitute MC conditions into FOC:
  - $\frac{n_{t+1}}{n_t(1+\sigma)} u'(n_{t+1}) = g'(n_t)$
  - $\Rightarrow \frac{n_{t+1}}{1+\sigma} u'(n_{t+1}) = n_t g'(n_t)$
  - The above says that the difference equation determining labor supply depends on $\sigma \to$ money not neutral here!

- Monetary steady state:
  - Do some comparative statics on the differences equation determining labor supply and get $\frac{\partial n}{\partial \sigma}$
  - In steady state:
    * $n_t = \bar{n}, \forall t$
    * $\Rightarrow \frac{1}{1+\sigma} u'(\bar{n}) = g'(\bar{n})$
    * $\Rightarrow \bar{n}(\sigma)$ (use IFT: $G(\sigma, n) = \frac{1}{1+\sigma} u'(\bar{n}) - g'(\bar{n}) = 0$ - or you can totally differentiate the function)
    * $\frac{\partial \bar{n}}{\partial \sigma} = \frac{u'(\bar{n})}{(1+\sigma)(u''(\bar{n}) - (1+\sigma)g''(\bar{n}))} = \frac{g'(\bar{n})}{u''(\bar{n}) - (1+\sigma)g''(\bar{n})} < 0$ (numerator is positive and denom negative).
    * This is the inflation tax - higher inflation induces people to work less since after "tax" wage less.
    * σ has an effect on the real economy, $\bar{n}$
  - What can we say about welfare?
    * Planner sets $u'(n^*) = g'(n^*) \Rightarrow \bar{n}(0) = n^*$
    * Thus the solution to the planners problem only obtains in the C.E. for $\sigma = 0$
    * Inflation tax is undesirable - first best outcome is $\sigma = 0$

Recall:

- FOC of OG model w/ money: $E\{\frac{p_t x_{t+1}}{p_{t+1}} u'\left(\frac{p_t n_t x_{t+1}}{p_{t+1}}\right)\} = g'(n_t)$
- Define $\rho \equiv \frac{p_t x_{t+1}}{p_{t+1}}$ as the stochastic real wage
- $\rho^e \equiv E(\frac{p_t x_{t+1}}{p_{t+1}}) = p_t E(\frac{x_{t+1}}{p_{t+1}})$, $n_t \sim n(\rho^e)$
- Then can write the FOC as: $\rho^e E u'(\rho^e n(\rho^e)) = g'(n(\rho^e))$
- $\rho^e E u'(\cdot) + cov(\rho^e, u') = g'(\cdot)$ (Which we get using the rule for the expected value of a product of random variables)

- $\frac{\partial\left(\frac{p_t x_{t+1}}{p_{t+1}}\right)}{\partial M_t} = \frac{\partial \rho^e}{\partial M_t} = 0 \Rightarrow$ neutrality of money
- SREE: $n(\rho^e) = \bar{n}, \rho = 1$, $p_t = p(M_t) = \frac{M_t}{\bar{n}}$
- DRAW two graphs. both have vertical axis at p_t and horizontal axis at y_t. In left graph have labor supply curve as vertical line at $\bar{n}$. In right graph have labor supply curve be upward sloping function $S(\rho^e)$. In first show that in $M_t \uparrow$ then p_t increase and the only reason for the increase in p_t is the increase in M_t. In the graph on the right, if $M_t \uparrow$ then move out along $S(\rho^e)$ and agent can't determine if increase in p_t driven by the increase in M_t or ρ^e.

Neutrality of money	Money not neutral
$corr(\$, y) = corr(\$, n) = 0$	$corr(\$, y) > 0,\ corr(\$, n) > 0 \rightarrow$ doesn't imply causality
$\frac{\partial \frac{p_t x_{t+1}}{p_{t+1}}}{M_t} = 0$	Need: 1) $n'(\rho^e) > 0 \rightarrow$ gross subs 2) $\frac{\partial \rho^e}{\partial M_t} > 0$
$\rho = 1 \rightarrow$ never change return to work with money	Device: 1) $M_t \uparrow$ 2) $p_t \uparrow$ (see this - could be $M_t \uparrow$ or something else) 3) Agent think $\rho^e \uparrow$ 4) $M_t \uparrow$ not observed (imperfect info)

Imperfect Information Model

- OG Model w/ production: $u(c_{t+1}) - g(n_t)$
- Lucas: $u(c_t^o, c_t^y, n_t)X \rightarrow$ we simplify to $u(c_t^o, n_t)$
- shocks:
  - \$ shocks - proportional transfers to old people (i.e., $M_{t+1} = M_t x_{t+1}$)
  - real shocks: demographic: 2 islands total population $=N_t$=1. Population of Island 1 $= \frac{\theta_t}{2}$, Pop of island 2=$\frac{1-\theta_t}{2}$ in period t.
  - Thus the shocks are x_t and θ_t
  - Both shocks drawn from known iid distributions
  - NOTE that money shocks (w/ non-neutrality) cause the sectors (islands) to move together, a pop shock causes them to move apart (b/c island 1 pop $= \frac{\theta_t}{2}$ and island 2 pop $= 1 - \frac{\theta_t}{2} \Rightarrow \theta_t \uparrow \Rightarrow$ pop 1 $\uparrow$, pop 2 $\downarrow$)
- Recall: $\rho = \frac{p_t x_{t+1}}{p_{t+1}}, \rho^e \equiv E\rho = E(\frac{p_t x_{t+1}}{p_{t+1}})$
  - $x_t \uparrow \Rightarrow M_t \uparrow \Rightarrow p_t \uparrow \Rightarrow M_{t+1} \uparrow \Rightarrow p_{t+1} \uparrow \Rightarrow$ ...
  - So an increase in the money supply in period t has a permanent effect
  - b/c of this, $p_{t+1} \uparrow$ so money neutral $\rightarrow$ no real response
- What about the real shock?
  - Island 1: $\theta_t \downarrow \Rightarrow p_t \uparrow$, but this does not imply $p_{t+1} \uparrow$
    * NOTE: prices move in opposite direction of population - less people (output) with same money supply means higher prices
    * shock to the population doesn't have a lasting effect
    * it just means less workers that one period, so prices rise for that period only

        * i.e., there is just a temporary effect
        * But this shock does induce a real response

- In this model, you observe $p_t \uparrow$, but you don't know the cause $\rightarrow$ could be a low population shock or an increase in the money supply
    - The response will be a convex combination of the responses to x_t and θ_t
    - i.e., observe $p_t \uparrow$ so produce some more, bust not as much as if know $\theta_t \downarrow$
- Initial Condition:
    - M_1 split equally across the two islands
    - $\Rightarrow M_t$ is the same across islands $\forall t$
- Information of gen. t agent:
    - know: M_{t-1}, p_t on my island
    - Don't know:
        * p_t on other island (if did, could solve for actual shock)
        * x_t, x_{t+1}
        * θ_t, θ_{t+1} (NOTE: can take expectations of these iid random variables)
        * $p_{t+1} \rightarrow$ can't take expectation of $\rightarrow$ need to devise SREE consistent with model

The Lucas Island Model

- $t = 1, 2, ...$
- 2 period lived agents
- preferences: $u(c_{t+1}) - g(n_t)$
- money supply: $M_{t+1} = M_t x_{t+1}$
    - x_t is iid - nominal shock to the economy
    - x_t is not island specific - each island has the same increase - so that money supply in two islands the same for all time
- Islands:
    - Fraction $\frac{\theta_t}{2}$ young agents born on island 1 in period t
    - $\Rightarrow 1 - \frac{\theta_t}{2}$ young agents born on island 2 (b/c total pop = 1)
    - θ_t is iid and uncorrelated w/ x_t
    - No interaction across islands
    - M_t is the same on each island $= \frac{M_t}{2}$
    - perfectly symmetric islands - same preferences, same technology, only diff is pop size
- Population: $N_t = 1$, but diff. fractions on the 2 islands
- Information:
    - Young generation t agents:

Know

- structure of the economy
- They are "solving the model"
- p_t
- M_{t-1} (money supply prior to t-period shock

Don't know

- (x_t, θ_t) - don't know today's shocks
- (x_{t+1}, θ_{t+1})
- p_{t+1}
- p_t on other island
- know nothing about the other island

- Question: How can we understand the correlation between money and real stuff? $\rightarrow corr(\$, \text{real})$
  - $corr(x, n) \neq 0, > 0$
  - that is, correlation between money shock and labor supply is not zero - in particular, when can you increase the money supply and make the whole economy (not just one island) grow?
- DRAW: SREE with arrows to: Ind opt, functions (which include beliefs of endogenous vars), market clearing
- Individual optimization:
  - For generation t young on island 1 (but remember, both islands are the same):
  - $\max_n E_{(x_{t+1}, p_{t+1} | p_t, M_{t-1})} u\left(\frac{p_t n_t x_{t+1}}{p_{t+1}}\right) - g(n_t)$
  - FOC w.r.t. n: $E_{(\cdot)}\{\left(\frac{p_t x_{t+1}}{p_{t+1}}\right) u'\left(\frac{p_t n_t x_{t+1}}{p_{t+1}}\right)\} = g'(n_t)$
- Market clearing, period t (for island 1):
  - $\underbrace{\frac{M_t}{2}}_{\text{supply}} = \underbrace{p_t n_t \frac{\theta_t}{2}}_{\text{demand}}$ (money market)
  - $\Rightarrow M_{t-1} x_t = p_t n_t \theta_t$
  - $\Rightarrow M_{t-1} = \frac{p_t n_t \theta_t}{x_t} \Rightarrow M_{t-1} = p_t n_t \left(\frac{\theta_t}{x_t}\right)$ (NOTE: that we know M_{t-1} and p_t and we choose n_t.)
- SREE
  - $p_t = p(M, x, \theta)$, $M \equiv$ inherited \$ - money before nominal shock (i.e. M_{t-1})
    - NOTE: no subscripts b/c stationary
    - $x \equiv$ current x
    - $\theta \equiv$ current θ
  - $y_t = n_t = n(M, x, \theta)$
- Lucas Conjecture:
  - $p(M, x, \theta) = M * \phi(z)$, where $z \equiv \frac{x}{\theta}$
  - $y = n(M, x, \theta) = \psi(z)$
  - Note: this is a guess at a SREE
    - p is proportional to M, n is independent of M (i.e., inherited money is neutral)
    - summarize (x, θ) by $z \equiv \frac{x}{\theta} \Rightarrow$ the guess above says everything just depends on the ratio of the shocks
  - Some examples:
    - $n\left(\frac{x}{\theta}\right)$, suppose $x \in \{0.8, 1.2\}$ and $\theta \in \{0.5, 1.5\}$, then $\frac{x}{\theta} \in \{\frac{0.8}{1.5}, \frac{1.2}{1.5}, \frac{0.8}{0.5}, \frac{1.2}{0.5}\}$
    - Here, given z we can find x and θ because they can only take on limited values $\Rightarrow$ money is neutral!

    - if $x \in \{0.8, 1.2\}$ and $\theta \in \{0.8, 1.2\}$, then $\frac{x}{\theta} \in \{\frac{0.8}{1.2}, 1, \frac{1.2}{0.8}\}$
    - $\Rightarrow$ can't find x from z and so money is not neutral!
  - In general: $p \to z \to (x, \theta)$ (note first arrow is reveals, second stage is infers)
    - If we can infer exactly x and θ, then we have a revealing equilibrium (in this case, money is neutral)
    - In general, we will work with non-revealing equilibria

- Steps to solving the problem:
  - Use functions from the guess in the MC condition, then in individual optimization (as for solving OG models)
  - Create and expression where $\psi(z)$is the only unknown $\to \psi(z)$ is a functional equation (F.E.)

- <u>Market clearing</u>:
  - $M_t = p_t n_t \theta_t \implies M_{t-1} x_t = p_t n_t \theta_t$
  - $\implies Mx = \underbrace{M * \phi(z)}_{p} * \underbrace{\psi(z)}_{n} * \theta, \forall (M, x, \theta)$ (this is the same as the previous MC, but no subscripts b/c stationary)
  - $\Rightarrow \phi(z) = \frac{z}{\psi(z)}, \forall (x, \theta)$ (Note that this is the connection between price and output (and notice no M here)

- To find SREE: Substitute MC into FOC:
  - FOC: $E_{(\cdot)}\{\frac{p_t x_{t+1}}{p_{t+1}} u'\left(\frac{p_t n x_{t+1}}{p_{t+1}}\right)\} = g'(n)$
  - $\frac{p_t x_{t+1}}{p_{t+1}} = \frac{\overbrace{M * \phi(z)}^{p_t} * x'}{\underbrace{M * x * \phi(z')}_{p_{t+1}}} = \frac{1}{\theta(\psi(z))} * \theta' \psi(z')$ (note $x' \equiv$ future x, and using new notation here)
  - NOTE: $\phi(z) = \frac{\frac{x}{\theta}}{\psi(z)} \Rightarrow \frac{\phi(z)}{x} = \frac{1}{\theta \psi(z)}$
  - with new notation: $E_{(\theta', x', \theta | z)}\{\frac{\theta' \psi(z')}{\theta \psi(z)} u'\left(\frac{\theta' \psi(z')}{\theta}\right)\} = g'(\psi(z)), \forall z$
  - OR $E_{(\theta', x', \theta | z)} U\left(\frac{\theta' \psi(z')}{\theta}\right) = G(\psi(z)), \forall z$ (*), where $U(c) \equiv cu'(c), G \equiv ng'(n)$ (remember that we don't know θ)
  - Comments about (*):
    - Know $p = M\phi(z)$ <u>not</u> z
    - Prove $\phi(z)$ strictly increasing $\Rightarrow p$ reveals z ($z_1 \neq z_2$ <u>but</u> $\phi(z_1) = \phi(z_2)$, but M.C. won't hold if p doesn't change with z (so it must)
    - Stationary notation: $E_{(\theta', x', \theta | z)} U\left(\frac{\theta' \psi(z')}{\theta}\right) = G(\psi(z)), \forall z$
    - the unknown in (*) is $\psi(z) \to$ This stationary function is what you are solving for!
    - Same $\psi(z)$ for each sector/island - they just have different arguments.
      - $z = \frac{x}{\theta}$ breaks the classical dichotomy b/c labor and consumption depend on $\psi(z)$, a function of z, which is the ratio of x and $\theta \Rightarrow$ labor, consumption depend on x

- Special cases:
  1. $\theta = 1$ w/ prob 1 $\Rightarrow$ the OG model w/ stochastic, proportional transfers
     - $E_x U(\psi(x)) = G(\psi(x)), \forall x$
     - a sol'n is $\psi(x) = \bar{n}, \forall x$ (we know this from solving it before - money is neutral)
     - $\bar{n}$ solves $u'(\bar{n}) = g'(\bar{n})$ from FOC

2. $x = 1$ w/ prob 1 $\Rightarrow$ OG model with stochastic population growth
   - $E_{\theta'}U\left(\frac{\theta'\psi(\frac{1}{\theta'})}{\theta}\right) = G(\psi(\frac{1}{\theta})), \forall\theta$
   - If $U(c)$ displays gross substitutes: $(U'(c) > 0)$, then $\theta \uparrow \Rightarrow \psi(\cdot) \downarrow$
     * See this: if $\theta \uparrow \Rightarrow LHS \downarrow$ b/c U increasing and θ in denominator of argument $\Rightarrow$ since equality must hold, $RHS \downarrow \Rightarrow \psi(\frac{1}{\theta}) \downarrow$ since $G'(n) > 0$
   - $\theta_t \uparrow \Rightarrow E\left(\frac{p_t}{p_{t+1}}\right) \downarrow$ as $p_t \downarrow$
   - $\downarrow p_t = M\phi(\frac{1}{\theta_t}) = M\left(\frac{(\frac{1}{\theta_t})}{\psi(\frac{1}{\theta_t})}\right) \rightarrow$ both numerator and denominator falling, so need to know if numerator decreases faster than the denominator to know if price and per capita output move in the same direction
   - DRAW graph. Vertical is p_t, horizontal is $output_t$. Have supply and demand curves. Show supply curve shifting out as increase θ_t. In this case more population = higher prices = more output b/c gross substitutes

- Example: (w/ Gross subs $\Rightarrow U'(\cdot) > 0$)
  - $\theta \in \{\theta_L, \theta_H\}$, w/ prob $\pi_L, \pi_H, \pi_L + \pi_H = 1$
  - <u>2 Equations</u>:
    1. $\pi_L U\left(\frac{\theta_L\psi(\frac{1}{\theta_L})}{\theta_H}\right) + \pi_H U\left(\frac{\theta_H\psi(\frac{1}{\theta_H})}{\theta_H}\right) = G(\psi(\frac{1}{\theta_H}))$
       * This yields the expected labor supply if in high
    2. $\pi_L U\left(\frac{\theta_L\psi(\frac{1}{\theta_L})}{\theta_L}\right) + \pi_H U\left(\frac{\theta_H\psi(\frac{1}{\theta_H})}{\theta_L}\right) = G(\psi(\frac{1}{\theta_H}))$
       * This yields the expected labor supply if in high
  - The above are two equations with two unknowns: $\psi(\frac{1}{\theta_L})$ and $\psi(\frac{1}{\theta_L})$
  - Claim: $\theta_L < \theta_H \Rightarrow \psi(\frac{1}{\theta_L}) > \psi(\frac{1}{\theta_H})$ (b/c $\psi(\cdot)$ increasing in $z = \frac{1}{\theta}$ b/c $x = 1$ w/ prob 1)
  - Proof: Clearly, LHS of 2) bigger than LHS of 1) (b/c $\theta_H > \theta_L$ and they being in denominator is the only diff between 1) and 2) - and we are in the gross substitutes case, so U is increasing)
    * $\Rightarrow G(\psi(\frac{1}{\theta_H})) < G(\psi(\frac{1}{\theta_L}))$
    * $\Rightarrow \psi(\frac{1}{\theta_L}) > \psi(\frac{1}{\theta_H})$ (b/c $G(\cdot)$ increasing $\Rightarrow \psi(\cdot)$ decreasing in θ.)

- Case with both shocks (x & θ)
  - $p = M\phi(z), z \equiv \frac{x}{\theta}$
  - Revealing: z reveals x and θ (e.g. $x \in \{0.5, 1.5\}, \theta \in \{0.8, 1.2\} \Rightarrow$ 4 values of z)
    * $\exists$ an SREE in which $n = \psi(z) = \tilde{\psi}(\theta) \rightarrow$ No x (b/c we can infer x)
  - Non-revealing
    * Know z, but not x and θ
    * Agents observe $p \uparrow$, but don't know the cause $\Rightarrow p = M\phi(z)$

- An SREE is characterized by:
  - (*) $\underbrace{H(z)}_{\text{only a func of } z} \equiv E_{(\theta',\theta,z'|z)}U\left(\frac{\theta'\psi(z')}{\theta}\right) = G(\psi(z)), \forall z$
  - Lucas assumes $Pr(\theta \leq \hat{\theta}|z)$ increases in z, $\forall\hat{\theta}$
  - $\Rightarrow \psi(z)$ is increasing in z
  - <u>Key result</u>: (*) Gross Subs + assumption $Pr(\theta \leq \hat{\theta}|z)$ is increasing in z $\forall\hat{\theta} \Rightarrow H'(z) > 0$
  - $\Rightarrow \psi(z)$ increasing in z

- Why? - Directly from (*)
  - By assumption, z bigger $\Rightarrow \theta$ likely to be lower, θ lower make $c \uparrow$ (b/c $c = \frac{\theta' \psi(z')}{\theta}$), $c \uparrow$ means $U \uparrow \Rightarrow G(\cdot) \uparrow$ and $G(\cdot)$ is an increasing function so $\psi(z)$ must increase $\Rightarrow \psi(z)$ increasing in z
  - What's happening is a convex combination of the two special cases.
- Who cares?
  - Well, the result $\Rightarrow$ money <u>not</u> neutral!
  - $x \uparrow \Rightarrow z \uparrow \Rightarrow \psi(z) \uparrow \Rightarrow$ more output in all sectors

- <u>Fundamental equation characterizing SREE</u>
  - $E_{\theta', x', \theta|z} U\left(\frac{\theta' \psi(z')}{\theta}\right) = G(\psi(z)), \forall z$
  - Note that $\{\theta|z\}$ is the key to the conditional expectation since θ is what depends on z, not θ' or x'
    - Here, $U(c) \equiv cu'(c), G(n) \equiv g'(n)$
    - We assume $U'(c) > 0 \rightarrow$ the condition for the gross substitutes case
  - Assume: (*) $Pr(\theta \leq \hat{\theta}|z)$ is increasing in $z \forall \hat{\theta}$
  - Gross subs + assumption (*) $\Rightarrow \psi(z)$ is increasing in z
    - G.S. $\Rightarrow U'(\cdot) \uparrow$ if $z \uparrow, G(\cdot) \uparrow$ if $\psi(z) \uparrow \Rightarrow \psi(z) \uparrow$ in z
  - Money is not neutral!
    - $x \uparrow \Rightarrow$ output expands on all islands (sectors)
  - Key elements:
    - Intertemporal substitution
      - When real return to work $\uparrow$, you work more
      - Intertemporal b/c work/consume in different periods
    - Confusion
      - Observe $p \uparrow$, don't know if it's from x or θ
      - Assumption (*) about behavior under confusion